

## Chapter 3: Algorithmic Simplification Techniques

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### Karnaugh Map

	A	
B	A'	A
B'	00 0	10 2
B	01 1	11 3

Two-variable truth table

- Group of 1 : 2 variables
- Group of 2 : 1 variable

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### Karnaugh Map

	AB			
	A'		A	
C'	000 0	010 2	110 6	100 4
C	001 1	011 3	111 7	101 5
	B'	B	B'	B

Three-variable truth table

- Group of 1 : 3 variables
- Group of 2 : 2 variables
- Group of 4 : 1 variable

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### Karnaugh Map Four-variable truth table

AB		A'		A		D'	
		0000 0	0100 4	1100 12	1000 8		
CD		C'		C		D	
		0001 1	0101 5	1101 13	1001 9		
C		B'		B		D'	
		0011 3	0111 7	1111 15	1011 11		
D		B'		B		D'	
		0010 2	0110 6	1110 14	1010 10		
		B'		B			

- Group of 1 : 4 variables
- Group of 2 : 3 variables
- Group of 4 : 2 variables
- Group of 8 : 1 variable

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### Truth table → algebraic expression

a	b	f
0	0	0
0	1	1
1	0	1
1	1	0

- $f$  is 1 if  $a=0$  AND  $b=1$  OR if  $a=1$  AND  $b=0$
- $f$  is 1 if  $a'=1$  AND  $b=1$  OR if  $a=1$  AND  $b'=1$
- $f$  is 1 if  $a'b=1$  OR if  $ab'=1$
- $f = a'b + ab'$

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### Truth table → algebraic expression

a	b	f
0	0	0
0	1	1
1	0	1
1	1	1

- $f$  is 1 if  $a=0$  AND  $b=1$  OR if  $a=1$  AND  $b=0$  OR if  $a=1$  AND  $b=1$
- $f$  is 1 if  $a'=1$  AND  $b=1$  OR if  $a=1$  AND  $b'=1$  OR if  $a=1$  AND  $b=1$
- $f$  is 1 if  $a'b=1$  OR if  $ab'=1$  OR if  $ab=1$
- $f = a'b + ab' + ab$
- $f = a + b$

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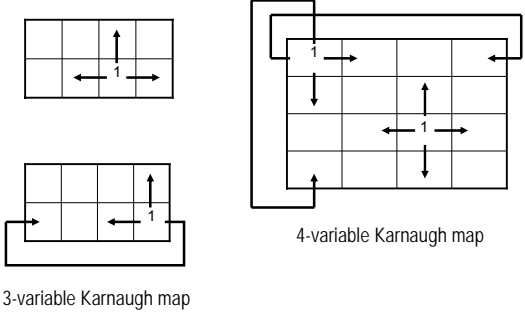
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## Adjacencies on 3- and 4- variable Karnaugh Maps




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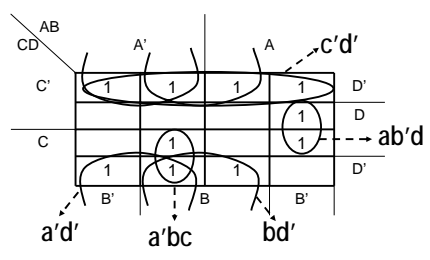
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## Example: $f(a,b,c,d) = \sum m(0,2,4,6,7,8,9,11,12,14)$



$$f = a'd' + bd' + a'bc + ab'd + c'd'$$

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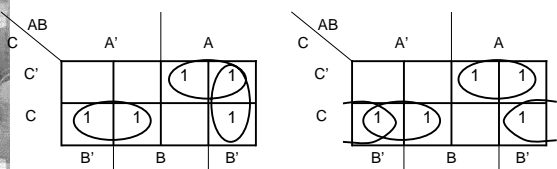
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## Two possible good answers



$$= A'C + AC' + AB'$$

$$= A'C + AC' + B'C$$

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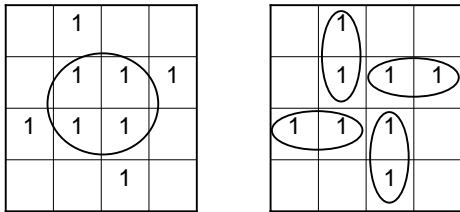
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## Don't be greedy!



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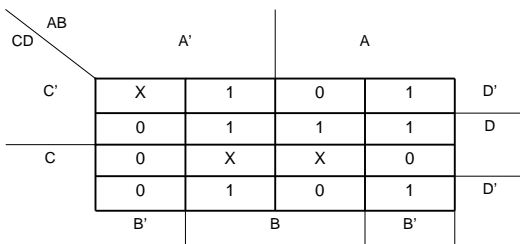
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## Don't Care



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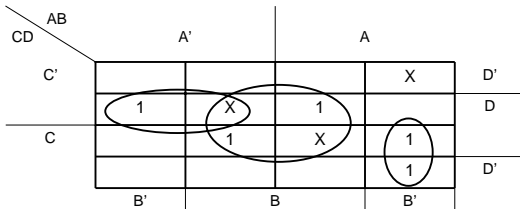
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## Example:

$$f(a,b,c,d) = \sum m(1,7,10,11,13) + \sum d(5,8,15)$$



'X' can be included in a group if it is useful or can be left out of the loops.

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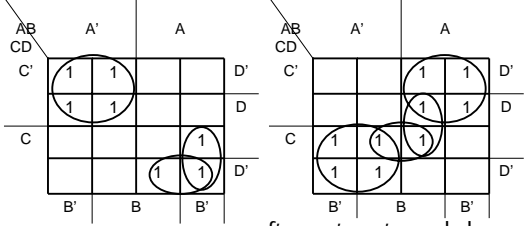
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Example:  $f(a,b,c,d) = \sum m(0,1,4,5,10,11,14)$

$\therefore f'(a,b,c,d) = \sum m(2,3,6,7,8,9,12,13,15)$



SOP  $\leftarrow f = a'c + ab'c + acd'$

$$f' = ac' + a'c + abd$$

$$= ac' + a'c + bcd$$

POS  $\left\{ \begin{aligned} f &= (a'+c)(a+c')(a'+b'+d') \\ &= (a'+c)(a+c')(b'+c'+d') \end{aligned} \right.$

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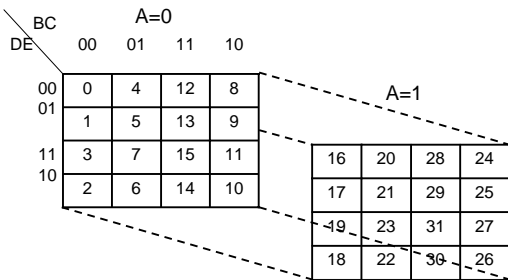
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### 5-variable Karnaugh Map




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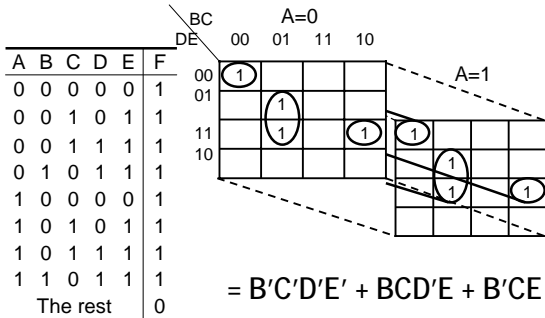
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### 5-variable Karnaugh Map (cont.)




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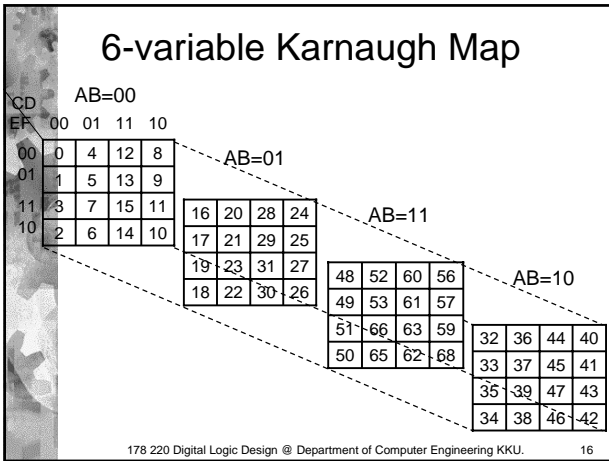
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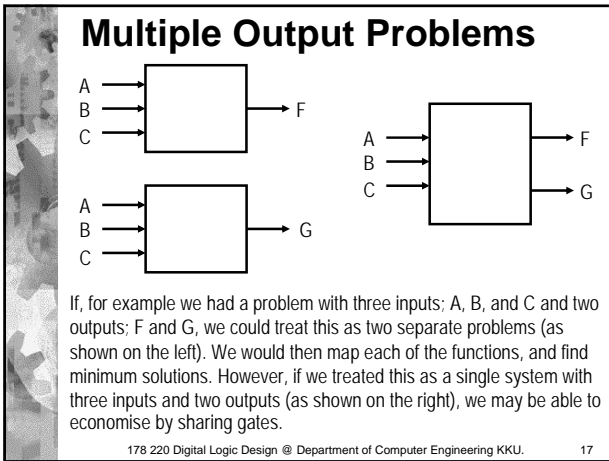
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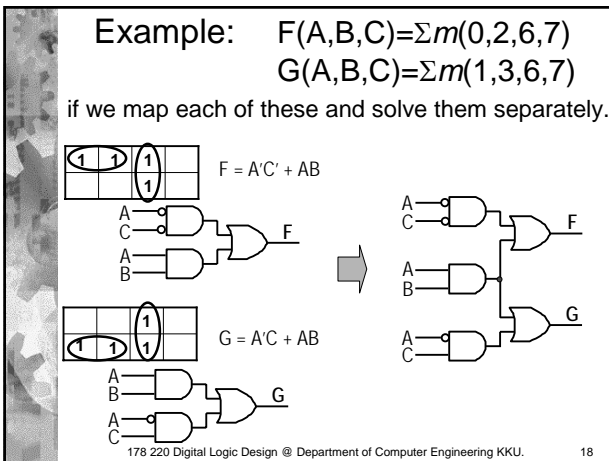
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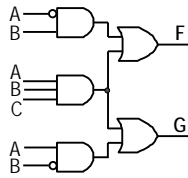
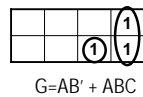
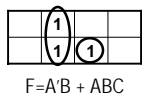
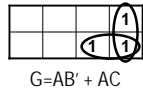
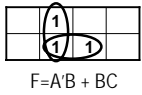
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Example:  $F(A,B,C)=\Sigma m(2,3,7)$   
 $G(A,B,C)=\Sigma m(4,5,7)$




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Not yet include section 3.2  
 An algorithmic minimisation technique

- 3.2.1 iterated consensus for one output
- 3.2.2 prime implicant tables for one output
- 3.2.3 iterated consensus for multiple output

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