Transformations

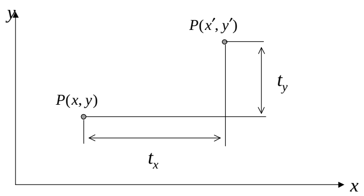
Transformations

- Alter position of a point
- Coordinate system
- modeling

Translation

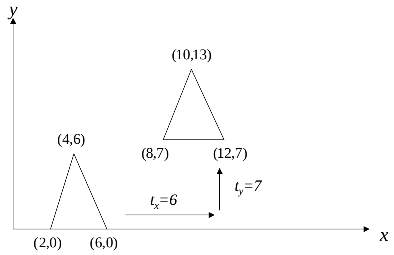
- Add *constant* corresponding to amount to be translated
- If constant is *positive*, point moves towards increasing values
- If constant is *negative*, point moves in direction of decreasing values
- Can add *separate* constant for each axis

Translation in 2-D



$$x' = x + t_x$$
$$y' = y + t_y$$

Translation in 2-D (concl'd)



Translation in 3-D

- Translation in three dimensions requires one additional equation

$$z' = z + t_z$$

Scaling (1 of 5)

- *Multiplication* by factor (constant) corresponding to amount to be scaled along axis
- If absolute value of factor *exceeds unity*, point moves *away* from origin
- If absolute value of factor *less than* unity, point moves *towards* origin
- If factor *negative*, point is *reflected* (and moved)
- Applying scaling to object moves it away from or towards origin
- To *avoid movement* of object, scaling must be relative to some *fixed point*

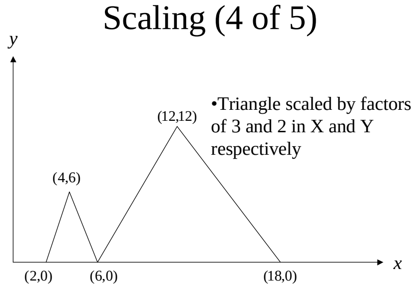
Scaling (2 of 5)

- *Scaling without movement* requires *translating* object so that fixed point becomes origin, then *scaling* it, then *translating* back
- What is *fixed point* ?
- *Eg*: tree-relative to ground position
- This is example of *concatenation* of transformations
- *Uniform scaling-same* scale factors on all axes
- *Differential scaling* – scaling on some axis (axes) *different* than on other axis (axes)

Scaling (3 of 5)

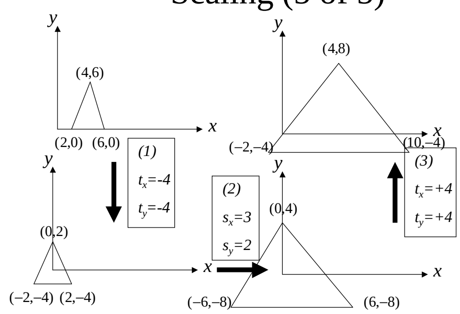
$$x' = s_x x$$
$$y' = s_y y$$

Scaling (4 of 5)



•Triangle scaled by factors of 3 and 2 in X and Y respectively

Scaling (5 of 5)

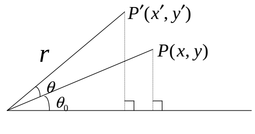


Scaling in 3-D

- Scaling in three dimensions requires one additional equation

$$z' = s_z z$$

Rotation



- θ is counter clockwise angle of rotation
- θ_0 is counter clockwise angle to *P(x,y)* in standard position (measured from horizontal line through origin)
- *r* is distance to *P* from origin (same as to *P'* from origin)

Rotation (cont'd)

$$\cos(\theta_0 + \theta) = x'/r$$
$$\cos\theta_0 \cos\theta - \sin\theta_0 \sin\theta = x'/r$$
$$\frac{x}{r}\cos\theta - \frac{y}{r}\sin\theta = x'/r$$
$$\underline{x' = x\cos\theta - y\sin\theta}$$

$$\sin(\theta_0 + \theta) = y'/r$$
$$\sin\theta_0 \cos\theta + \cos\theta_0 \sin\theta = y'/r$$
$$\frac{y}{r}\cos\theta + \frac{x}{r}\sin\theta = y'/r$$
$$\underline{y' = x\sin\theta + y\cos\theta}$$

Rotation (concl'd)

- Equations for coordinates of rotated point (*P'*) as explicit functions of angle and original coordinates

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

- Generalizing to rotation about (c_x, c_y)

$$x' = (x - c_x)\cos\theta - (y - c_y)\sin\theta$$
$$y' = (x - c_x)\sin\theta + (y - c_y)\cos\theta$$

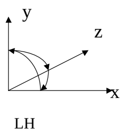
- Programming note: when computing *x'* retain *x* for use in second equation

Rotation in 3D (1 of 6)

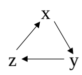
- Must specify axis of rotation and direction
- Direction can be specified as from positive *f*-axis towards positive *t*-axis

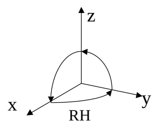
Rotation in 3-D (2 of 6)

- Table to define positive rotations
  - from positive x-axis to positive y-axis about z-axis
  - from positive y-axis to positive z-axis about x-axis
  - from positive z-axis to positive x-axis about y-axis



Axis of rotation	f-axis	t-axis
Z	X	Y
X	Y	Z
Y	Z	X





Rotation in 3-D (3 of 6)

- This is a logical generalization from 2D rotation which is from positive x-axis to positive y-axis

Rotation in 3D (4 of 6)

- About z-axis

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

- generalizing from *f*-axis to *t*-axis about *r*-axis:

$$f' = f\cos\theta - t\sin\theta$$
$$t' = f\sin\theta + t\cos\theta$$
$$r' = r$$

Rotation in 3D (5 of 6)

- About x-axis $(y \leftarrow f, z \leftarrow t, x \leftarrow r)$

$$x' = x$$
$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$

- About y-axis $(z \leftarrow f, x \leftarrow t, y \leftarrow r)$

$$x' = z\sin\theta + x\cos\theta$$
$$y' = y$$
$$z' = z\cos\theta - x\sin\theta$$

Rotation in 3-D (6 of 6)

- *Thumb rule* -- thumb pointing in direction of increasing positive values, fingers curl in direction of positive angle of rotation
- Use left hand for left handed coordinate system and right hand for right handed coordinate system

Compound Transformations (1 of 6)

- *Concatenate* several *simple* transformations to construct general transformations
- Result is a sequence of simple transformations called a *compound* transformation

Compound Transformations (2 of 6)

- *Ordering* in the sequence is important (e.g: 3D rotation about one axis followed by rotation about another axis differs from performing the two rotations in the opposite order)
- In some cases, ordering of the sequence is irrelevant (e.g.: translations, scaling, 2D rotations)

Compound Transformations (3 of 6)

- *Example*: under what conditions will a scale and rotation transformation commute ?
- Scale followed by a rotation about y

$$x' = (s_z z)\sin\theta + (s_x x)\cos\theta$$
$$y' = s_y y$$
$$z' = (s_z z)\cos\theta - (s_x x)\sin\theta$$

- rotation about y followed by a scale

$$x' = s_x(z\sin\theta + x\cos\theta) = (s_x z)\sin\theta + (s_x x)\cos\theta$$
$$y' = s_y y$$
$$z' = s_z(z\cos\theta - x\sin\theta) = (s_z z)\cos\theta - (s_z x)\sin\theta$$

- by observation, results are same if $s_x = s_z$

Compound Transformations (4 of 6)

- Transformations commute if scale factors are the same on the axes that are not the axis of rotation
- A useful special case is uniform scaling

Compound Transformations (5 of 6)

- Example: rotation about point (c_x,c_y) in 2D

1) translate center of rotation to origin

$$x' = x - c_x$$
$$y' = y - c_y$$

2) rotate about origin through angle θ

$$x'' = x'\cos\theta - y'\sin\theta$$
$$y'' = x'\sin\theta + y'\cos\theta$$

3) translate origin back to center of rotation

$$x''' = x'' + c_x$$
$$y''' = y'' + c_y$$

Compound Transformations (6 of 6)

- Substitute (1) into (2) into (3)

$$x''' = (x - c_x)\cos\theta - (y - c_y)\sin\theta + c_x$$
$$y''' = (x - c_x)\sin\theta + (y - c_y)\cos\theta + c_y$$