Order of Transformations

$$[x''' \quad y''' \quad 1] = [x'' \quad y'' \quad 1]M_3$$
$$= ([x' \quad y' \quad 1]M_2)M_3$$
$$= (([x \quad y \quad 1]M_1)M_2)M_3$$
$$= [x \quad y \quad 1]M$$

where $M = M_1 M_2 M_3$

Order of Transformations (cont'd)

- What about column vector convention ?

$$\begin{bmatrix} x''' \\ y''' \\ 1 \end{bmatrix} = N_3 \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = N_3 N_2 \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = N_3 N_2 N_1 \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = N \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Where $N = N_3 N_2 N_1$

Order of Transformations (Concl'd)

- Note that N shows matrices in reverse order
- Also note that:

$$N_i = M_i^{\,t}, i = 1,2,3$$

- Furthermore:

$$N = N_3 N_2 N_1$$
$$= M^t{}_3 M^t{}_2 M^t{}_1$$
$$= (M_1 M_2 M_3)^t$$
$$= M^t$$

Matrix Stack

- For row vectors

$$[x \quad y \quad 1]M_1 M_2 M_3$$

- To form compound matrix prior to multiplication by point, order of matrix multiplication is right to left

Matrix Stack (Cont'd)

- Thus, for "current" (accumulated) matrix C

$$C \leftarrow M_3 \quad (\Rightarrow C = M_3)$$
$$C \leftarrow M_2 C \quad (\Rightarrow C = M_2 M_3)$$
$$C \leftarrow M_1 C \quad (\Rightarrow C = M_1 M_2 M_3)$$

- i.e., Current transformation matrix is premultiplied by new transformation matrix
- Thus, calls to transformation routines are in reverse order from their desired effects
- Calls are in order M_3,M_2,M_1 to achieve the effect of transformations applied in order M_1,M_2,M_3

Matrix Stack (Cont'd)

- For column vectors

$$N_3 N_2 N_1 \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- To form compound matrix prior to multiplication by point, order of matrix multiplication is left to right

Matrix Stack (Cont'd)

- Thus, for "current" (accumulated) matrix C

$$C \leftarrow N_3 \quad (\Rightarrow C = N_3)$$
$$C \leftarrow CN_2 \quad (\Rightarrow C = N_3 N_2)$$
$$C \leftarrow CN_1 \quad (\Rightarrow C = N_3 N_2 N_1)$$

- i.e., Current transformation matrix is postmultiplied by new transformation matrix
- Thus, calls to transformation routines are still in reverse order from their desired effects

Matrix Stack (Concl'd)

- Calls in order N_3,N_2,N_1 to achieve the effect of transformation applied in order N_1,N_2,N_3