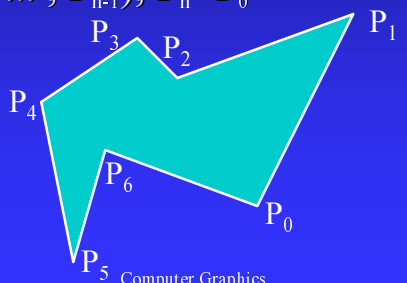
Geometry

Area of polygons &
Volume Of Polygonal surfaces

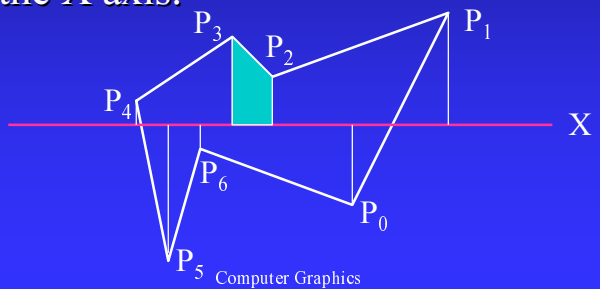
Calculating Area

- Calculation of the area of a polygon can be done in a very simple way.
- Assume that the polygon is represented by $(P_0, P_1, P_2, \ldots, P_{n-1})$, $P_n = P_0$



Step 1

- Consider the problem of calculating the area of the trapezoid that is generated between any 2 consecutive points P_i and P_{i+1} and the X axis.



Step 2

- Starting from P_0, Calculate the area of the trapezoid which is generated between P_i, P_{i+1} and the X axis. Assume $P_i=(X_i,Y_i)$ and $P_{i+1}=(X_{i+1}, Y_{i+1})$

$$V = \frac{1}{2}\cdot(X_{i+1} - X_i)\cdot(Y_{i+1} + Y_i)$$

- Add this area to the total volume sum.

Step 3

- Repeat Step 2 for each 2 consecutive points in the polygon.
- At the end of the process, the area will be calculated (up to the sign).
- Note the this algorithm works for all pairs (two positive, two negative and even one positive and one negative)!

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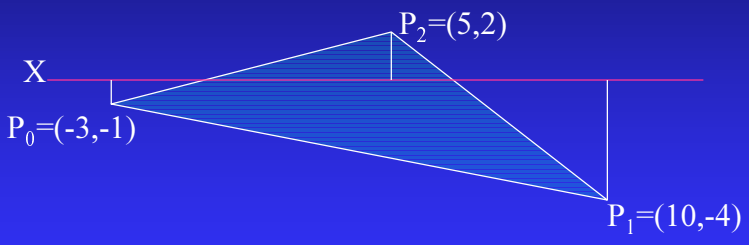
Example

- Let's calculate the area of a triangle.



V = (x1-x0)(y1+y0)/2+(x2-x1)(y2+y1)/2+(x0-x2)(y0+y2)/2
= |(10+3)(-4-1)/2 + (5-10)(2-4)/2 +(-3-5)(-1+2)/2| = 63/2

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Volume Of a Polygonal Surface

- A similar method can be used to calculate the volume of polygonal surfaces.
- We'll assume that the polygons in the surface are made of triangles.
- We will examine the volume that is generated between a single triangle and the x-z plane.

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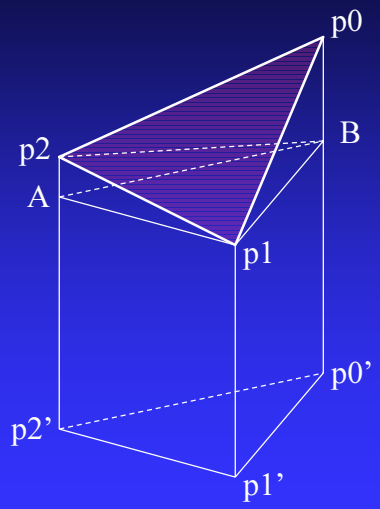
Volumes - Cont.



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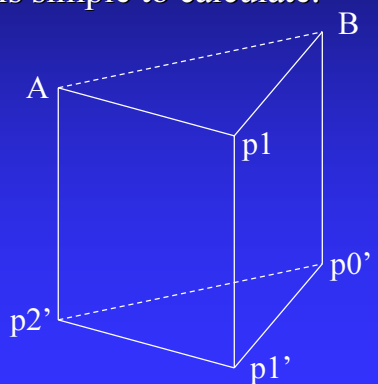
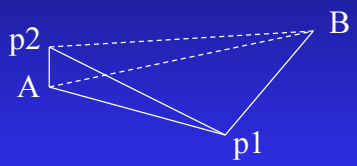
Volumes - Cont.

- Calculating the volume is done in 3 stages:
  - The volume of between A-B-p1 and p2'-p1'-p0' is simple to calculate.



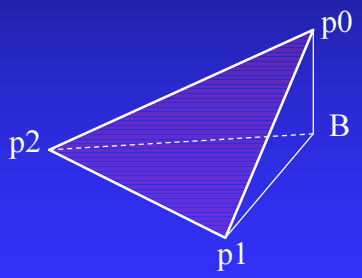
Volumes - Cont.

- Now, it's also simple to calculate the volume of the pyramid p2-B-p1-A



Volumes - Cont.

- And the final volume part is the pyramid p2-p0-B-p1, which is also easy to calculate



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Volumes - Cont.

- The sum of these 3 volumes provides us with the total volume between the polygon and the X-Z plane.
  - This volume may be negative! (depending on the location of the polygon in space)
- But hey! when do we add and when do we subtract different volumes?
  - We can use the normal of the polygon to decide whether it is facing the positive Y direction.
  - If it does - We add the volume to the total sum.
  - Otherwise we subtract this volume from the total volume sum.

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