Homework 4

188 200 Discrete Mathematics and Linear Algebra

Due: — Monday : August 24, 2009: All homework will be collected at 6:00pm. Any homework submitted after that are late.

Note:
1. All homework must be submitted to the submission box.
2. Submitting homework by email is no longer allowed.

1. Solve the following system of linear equations (i.e. find $x_1$, $x_2$ and $x_3$) by performing row reduced on the augmented matrix of the system.

\[
\begin{align*}
   x_1 + 2x_2 - 3x_3 &= 8 \\
   3x_1 + 2x_2 + 9x_3 &= 7 \\
   4x_1 + x_2 + 5x_3 &= -15
\end{align*}
\]

2. Find an equation involving $g$, $h$ and $k$ that makes the following augmented matrix correspond to a consistent system of linear equations. Show and explain each step of your work.

\[
\begin{bmatrix}
   1 & -4 & 7 & g \\
   0 & 3 & -5 & h \\
   -2 & 5 & -9 & k \\
\end{bmatrix}
\]

3. Row reduce the following matrix to reduced echelon form. List all pivots and pivot columns.

\[
\begin{bmatrix}
   1 & 3 & 5 & 7 \\
   3 & 5 & 7 & 9 \\
   5 & 7 & 9 & 1 \\
\end{bmatrix}
\]

4. Consider the following system of linear equations then answer questions a – d.

\[
\begin{align*}
   x_1 - 7x_2 + 6x_4 &= 5 \\
   x_3 - 2x_4 &= -3 \\
   -x_1 + 7x_2 - 4x_3 + 2x_4 &= 7
\end{align*}
\]

a. Write down the augmented form matrix of the system.

b. Is the system consistent? Justify your answer.

c. If your previous answer is “yes” then is the solution unique? Justify your answer.
d. Find the solution of the system.

5. Determine if \( \overrightarrow{b} \) is a linear combination of the vectors formed from the column of the following matrix \( A \). Explain your answer.

\[
A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \quad \quad \overrightarrow{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}
\]

6. Let \( \overrightarrow{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \), \( \overrightarrow{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix} \), \( \overrightarrow{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix} \). Find the value of \( h \) that makes \( \overrightarrow{y} \) in the plane spanned by \( \overrightarrow{v}_1 \) and \( \overrightarrow{v}_2 \).

7. Consider the following system of linear equations

\[
\begin{align*}
8x_1 - x_2 + 2x_3 &= 4 \\
5x_1 + 4x_2 + 7x_4 &= 11 \\
x_1 + 2x_2 + 3x_3 + 4x_4 &= 3
\end{align*}
\]

a. write the system as a vector equation
b. write the system as a matrix equation

8. Determine if the following matrix span \( \mathbb{R}^4 \). Explain your answer.

\[
\begin{bmatrix} 5 & -7 & -4 & 9 \\ 6 & -8 & -7 & 5 \\ 4 & -4 & -9 & -9 \\ -9 & 11 & 16 & 7 \end{bmatrix}
\]