1. Random Element & Random Variable

1.1 Define a Sample Space, Ω

In [6]:
```
Ω=Set(['H','T'])
show(Ω)
```

Sample function

In [95]:
```
set_random_seed(0) # Fixed sample function
n=10
Sample=[Ω.random_element() for i in range(n)];
Sample
```

Out[95]:
```
['H', 'H', 'H', 'T', 'H', 'H', 'H', 'T', 'H', 'T']
```

1.2 Random variable

a. Define a Function, $M : \Omega \to \mathbb{R}$

In [96]:
```
Ω=Set(['H','T'])
R=Set([1,0])
M = FiniteSetMaps(Ω, R)
M
```

Out[96]:
```
Maps from {'H', 'T'} to {0, 1}
```

In [97]:
```
M.domain()
M.codomain()
```

Out[97]:
```
{0, 1}
```

b. Define a Random variable ≡ Set mapping rules, X

In [98]:
```
X=M.from_dict({'H':1,'T':0})
X
```

Out[98]:
```
map: H -> 1, T -> 0
```

Sample function

In [99]:
```
#set_random_seed(0) # Fixed sample function
n=10
Sample=[]
for i in range(n):
    ω=Ω.random_element()
    Sample.append(X(ω))
Sample
```

Out[99]:
```
[1, 1, 1, 1, 1, 0, 0, 0, 0, 0]
```

Calculate the number of heads

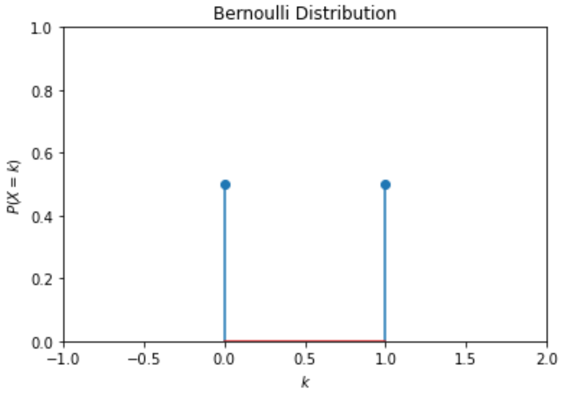
In [81]:
```
No_head=sum(Sample);No_head
```

Out[81]:
```
5
```

2. Importand Random variables

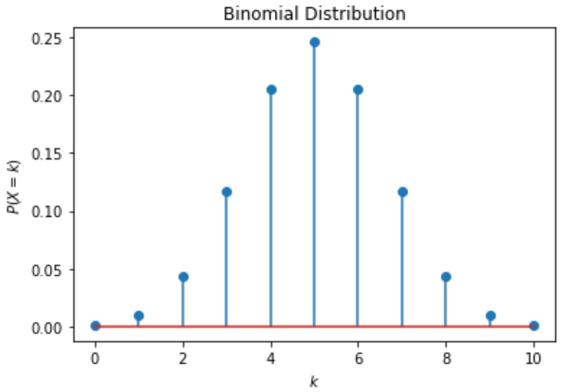
2.1 Bernoulli Ramdom variable

In [14]:
```
import numpy as np
from scipy.stats import bernoulli
import matplotlib.pyplot as plt
p=0.5
rv=bernoulli(p)
k=[i for i in np.arange(0,2)]
P=[rv.pmf(i) for i in np.arange(0,2)]
plt.stem(k, P)
plt.title("Bernoulli Distribution");
plt.xlabel('$k$');
plt.ylabel('$P(X=k)$');
plt.axis([-1,2,0,1])
plt.show()
```



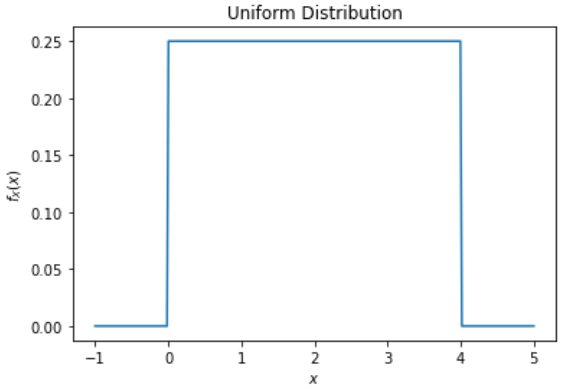
2.2 Binomial Ramdom variable

In [15]:
```
import numpy as np
from scipy.stats import binom
import matplotlib.pyplot as plt
N=10
p=0.5
binom_dist=binom(N,p)
k=[i for i in np.arange(0,N+1)]
P=[binom_dist.pmf(i) for i in np.arange(0,N+1)]
plt.stem(k, P)
plt.title("Binomial Distribution");
plt.xlabel('$k$');
plt.ylabel('$P(X=k)$');
plt.show()
```



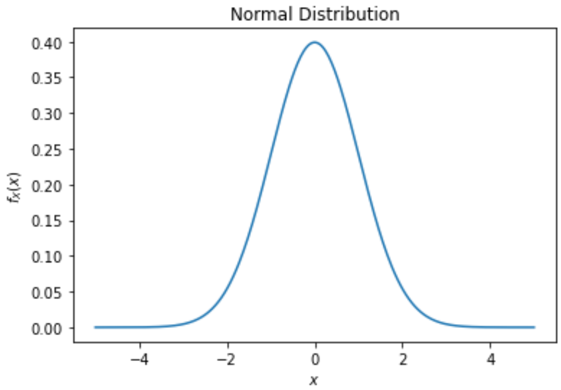
2.3 Uniform Random Variable

In [49]:
```
import numpy as np
from scipy.stats import uniform
import matplotlib.pyplot as plt
a=0
b=4
rv=uniform(loc=a,scale=b-a)
x = np.linspace(-1+a,1+b,300)
plt.plot(x,rv.pdf(x))
plt.title("Uniform Distribution");
plt.xlabel('$x$');
plt.ylabel('$f_X(x)$');
plt.show()
```



2.4 Gaussian Random Variable, $\mathcal{N}(\mu, \sigma)$

In [67]:
```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
μ=0
σ=1
rv=norm(loc=μ, scale=σ)
x = np.linspace(-5+μ,5+μ,300)
plt.plot(x,rv.pdf(x))
plt.title("Normal Distribution");
plt.xlabel('$x$');
plt.ylabel('$f_X(x)$');
plt.show()
```



3. Example 1

In [77]:
```
x,y=var("x,y",domain="real")
π=pi
f(x,y)=1/2/π*e^(-(2*x^2-2*x*y+y^2)/2)
show(f(x,y))
```

Proof $f(x, y) \geq 0$

In [78]:
```
bool(f(x,y)>=0)
```

Out[78]: True

Proof $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$

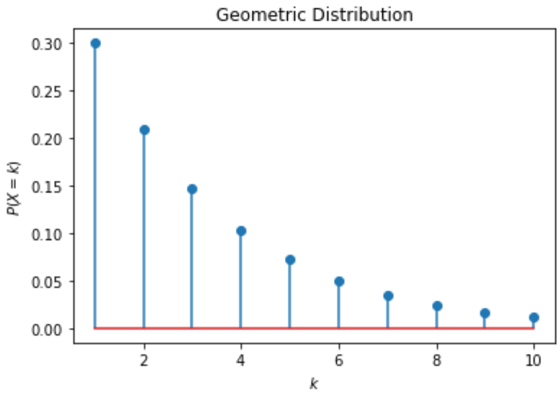
In [79]:
```
integral(integral(f(x,y),x,-oo,oo),y,-oo,oo)
```

Out[79]: 1

4. Addition Random variables

4.1 Geometric Random Variable

In [93]:
```
import numpy as np
from scipy.stats import geom
import matplotlib.pyplot as plt
N=10
p=0.3
rv = geom(p)
k=np.arange(1,N+1)
P=[rv.pmf(i) for i in np.arange(1,N+1)]
plt.stem(k, P)
plt.title("Geometric Distribution");
plt.xlabel('$k$');
plt.ylabel('$P(X=k)$');
plt.show()
```



5. Example 3 (only integrate part)

a. pdf of $X \sim \text{Uniform}(0, 1)$

In [104]:
```
f(x)=heaviside(x)-heaviside(x-1)
```

b. Calculate $P_Y(2)$

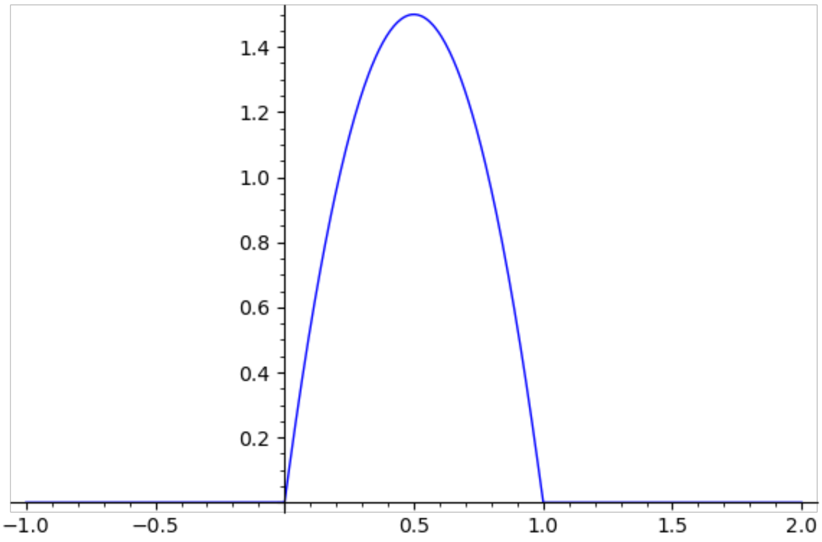
In [106]:
```
Py=integral(x*(1-x)*f(x),x,-oo,oo);Py
```

Out[106]: 1/6

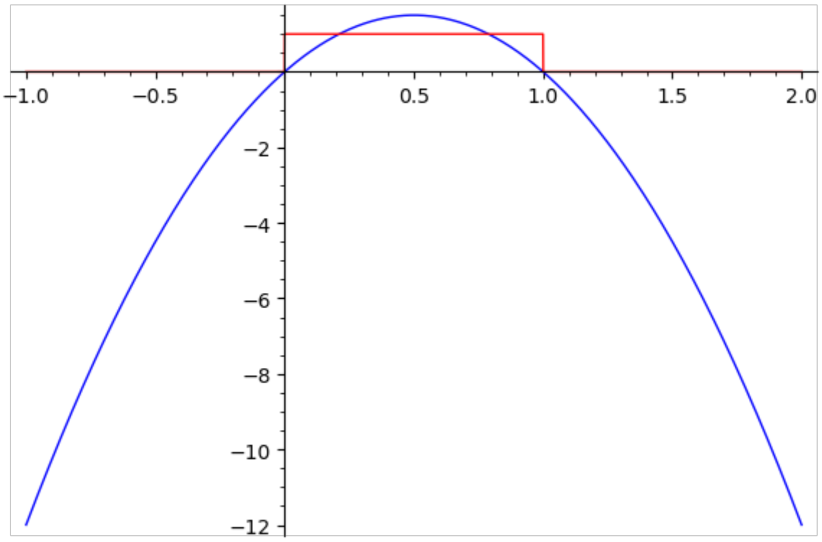
c. Plot $f_{X|Y}(x|2)$

In [112]:
```
show(x*(1-x)*f(x)/Py)
plot(x*(1-x)*f(x)/Py,x,-1,2)
```

Out[112]:



In [111]:
```
P1=plot(x*(1-x)/Py,x,-1,2)
P2=plot(f(x),x,-1,2,color="red")
P=P1+P2
show(P)
```



6. Example 4

a. define $f_X(x)$

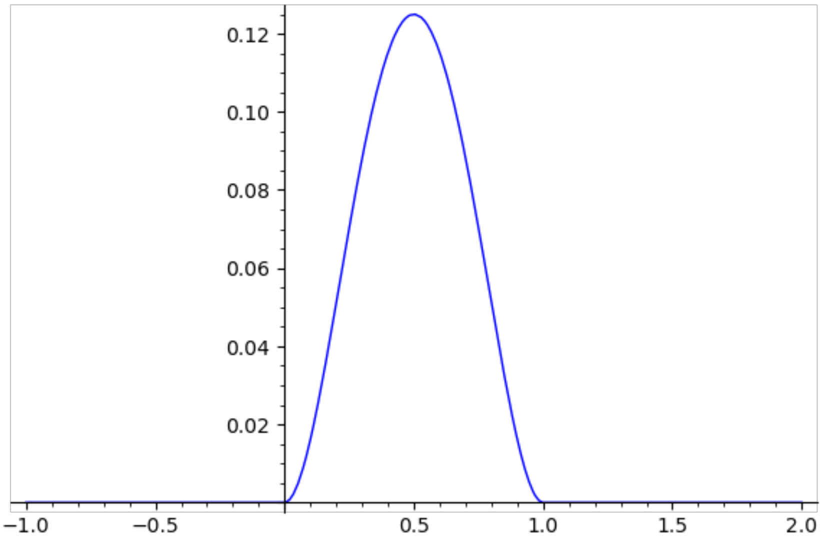
In [7]:
```
x=var("x")
f(x)=2*x*(heaviside(x)-heaviside(x-1))
```

b. calculate, $P_{Y|X}(3|x)f_X(x)$

In [8]:
```
Pf(x)=x*(1-x)^2*f(x)
print(Pf(x))
plot(Pf(x),x,-1,2)
```

```
-2*(x - 1)^2*x^2*(heaviside(x - 1) - heaviside(x))
```

Out[8]:



c. find $\hat{x}_{\mathrm{MAP}}$

In [9]:
```
d=diff(2*(x-1)^2*x^2,x);show(d)
show(solve(d,x))
```

$$4\,(x-1)^2x+4\,(x-1)x^2$$

$$\left[x=\left(\frac{1}{2}\right), x=0, x=1\right]$$

d.Test x

In [10]:
```
show([Pf(1/2),Pf(0),Pf(1)])
```

$$\left[\frac{1}{8}, 0, 0\right]$$

In [ ]: