Solution to Homework 1

188 200 Discrete Mathematics and Linear Algebra

Section 1.1

5.b (5) Not a statement, because the truth value of the sentence depends on “she”.

5.c (5) It is a statement, because the sentence is false.

5.d (5) Not a statement, because the truth value of the sentence depends on the variable x.

8.b (5) $\neg w \wedge (h \wedge s)$

8.c (5) $\neg w \wedge \neg h \wedge \neg s$

13 (10)

P	Q	$P \wedge Q$	$P \vee Q$	$\neg(P \vee Q)$	$(P \wedge Q) \vee \neg(P \vee Q)$
T	T	T	T	F	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	T

15 (10)

P	Q	R	$\neg P$	$\neg R$	$Q \vee \neg R$	$\neg P \wedge (Q \vee \neg R)$
T	T	T	F	F	T	F
T	T	F	F	T	T	F
T	F	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

18 (10)

P	Q	$P \vee Q$	$\neg P$	$\neg Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	T

Because the last two columns have the same truth values, $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.

20 (10)

P	$\mathbf{c}$	$P \vee \mathbf{c}$	P
T	F	T	T
F	F	F	F

Because the last two columns have the same truth values, $(P \vee \mathbf{c}) \equiv P$.

39 (10)

P	Q	R	$\neg P$	$\neg Q$	$\neg P \wedge Q$	$Q \wedge R$	$(\neg P \wedge Q) \wedge (Q \wedge R)$	$((\neg P \wedge Q) \wedge (Q \wedge R)) \wedge \neg Q$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T	F
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	F	F	F	F

Because the last column shows that all truth values of $((\neg P \wedge Q) \wedge (Q \wedge R)) \wedge \neg Q$ are false, $((\neg P \wedge Q) \wedge (Q \wedge R)) \wedge \neg Q$ is a contradiction.

44 (10)

$$\begin{aligned}
P \wedge (\neg Q \vee P) &\equiv P \wedge (P \vee \neg Q) && \text{the commutative law for } \vee \\
&\equiv (P \vee \mathbf{F}) \wedge (P \vee \neg Q) && \text{the identity law} \\
&\equiv P \vee (\mathbf{F} \wedge \neg Q) && \text{the distributive law} \\
&\equiv P \vee \mathbf{F} && \text{the domination law} \\
&\equiv P && \text{the identity law}
\end{aligned}$$

45 (10)

$$\begin{aligned}
\neg(P \vee \neg Q) \vee (\neg P \wedge \neg Q) &\equiv (\neg P \wedge \neg(\neg Q)) \vee (\neg P \wedge \neg Q) && \text{the De Morgan's law} \\
&\equiv (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) && \text{the double negation law} \\
&\equiv \neg P \wedge (Q \vee \neg Q) && \text{the distributive law} \\
&\equiv \neg P \wedge \mathbf{T} && \text{since } Q \vee \neg Q \equiv \mathbf{T} \\
&\equiv \neg P && \text{the identity law}
\end{aligned}$$

Problem 1 (10)

$$\begin{aligned}
P \leftrightarrow Q &\equiv (P \to Q) \wedge (Q \to P) && \text{the property of bi-conditional statement} \\
&\equiv (\neg P \vee Q) \wedge (\neg Q \vee P) && \text{the property of conditional statement} \\
&\equiv [(\neg P \vee Q) \wedge \neg Q] \vee [(\neg P \vee Q) \wedge P] && \text{the distributive laws} \\
&\equiv [(\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)] \vee [(\neg P \wedge P) \vee (Q \wedge P)] && \text{the distributive laws} \\
&\equiv [(\neg P \wedge \neg Q) \vee \mathbf{F}] \vee [\mathbf{F} \vee (Q \wedge P)] && \text{since } P \wedge \neg P \equiv \mathbf{F} \text{ and } Q \wedge \neg Q \equiv \mathbf{F} \\
&\equiv (\neg P \wedge \neg Q) \vee (Q \wedge P) && \text{the identity law} \\
&\equiv (P \wedge Q) \vee (\neg P \wedge \neg Q) && \text{the commutative law}
\end{aligned}$$

Problem 2 (10)

$$\begin{aligned}
[\neg Q \wedge (P \to Q)] \to \neg P &\equiv \neg(\neg Q \wedge (P \to Q) \vee \neg P && \text{the property of conditional statement} \\
&\equiv (Q \vee \neg(P \to Q) \vee \neg P && \text{the double negation law} \\
&\equiv (Q \vee \neg(\neg P \vee Q) \vee \neg P && \text{the property of conditional statement} \\
&\equiv (Q \vee (P \wedge \neg Q) \vee \neg P && \text{the double negation law} \\
&\equiv [(Q \vee P) \wedge (Q \vee \neg Q)] \vee \neg P && \text{the distributive law} \\
&\equiv [(Q \vee P) \wedge \mathbf{T}] \vee \neg P && \text{since } Q \vee \neg Q \equiv \mathbf{T} \\
&\equiv (Q \vee P) \vee \neg P && \text{the identity law} \\
&\equiv Q \vee (P \vee \neg P) && \text{the associative law} \\
&\equiv Q \vee \mathbf{T} && \text{since } P \vee \neg P \equiv \mathbf{T} \\
&\equiv \mathbf{T} && \text{the domination law}
\end{aligned}$$

Section 1.3

9 (10)

Premise: $P \to Q$ and $P \to R$

Conclusion: $P \to Q \wedge R$

Row	P	Q	R	$Q \wedge R$	$P \to Q$	$P \to R$	$P \to (Q \vee R)$
1	T	T	T	T	**T**	**T**	**T**
2	T	T	F	F	T	F	F
3	T	F	T	F	F	T	F
4	T	F	F	F	F	F	F
5	F	T	T	T	**T**	**T**	**T**
6	F	T	F	F	**T**	**T**	**T**
7	F	F	T	F	**T**	**T**	**T**
8	F	F	F	F	**T**	**T**	**T**

At row 1, 5, 6, 7 and 8, when both premises are true, the conclusion is true. Therefore the argument is valid.

10 (10)

Premise: $P \wedge \neg Q \to R$ and $P \vee Q$ and $Q \to P$

Conclusion: R

Row	P	Q	R	$\neg Q$	$P \wedge \neg Q$	$P \wedge \neg Q \to R$	$P \vee Q$	$Q \to P$	R
1	T	T	T	F	F	**T**	**T**	**T**	**T**
2	T	T	F	F	F	**T**	**T**	**T**	**F**
3	T	F	T	T	T	**T**	**T**	**T**	**T**
4	T	F	F	T	T	F	T	T	F
5	F	T	T	F	F	T	T	F	T
6	F	T	F	F	F	T	T	F	F
7	F	F	T	T	F	T	F	T	T
8	F	F	F	T	F	T	F	T	F

At row 1,2 and 3, all premises are true. However at row 2 the conclusion is false. Therefore the argument is not valid.

22 (10)

Let P be "Oleg is a math major".
Q be "Oleg is an economics major"
R be "Oleg is required to take Math 362"

The argument are

$P \vee Q$

$P \to R$

$\therefore Q \vee \neg R$

Row	P	Q	R	$\neg R$	$P \vee Q$	$P \to R$	$Q \vee \neg R$
1	T	T	T	F	**T**	**T**	**T**
2	T	T	F	T	T	F	T
3	T	F	T	F	**T**	**T**	**F**
4	T	F	F	T	T	F	T
5	F	T	T	F	**T**	**T**	**T**
6	F	T	F	T	**T**	**T**	**T**
7	F	F	T	F	F	T	F
8	F	F	F	T	F	T	T

At rows 1, 3, 5 and 6 both premises are true. But the conclusion is false at row 3. Therefore the argument is invalid.

41 (15)

$Q \to R$	Premise	(1)
$\neg R$	Premise	(2)
$\neg Q$	Modus tollens from (1) and (2)	(3)
$P \vee Q$	Premise	(4)
P	Disjunctive syllogism from (3) and (4)	(5)
$\neg Q \to U \wedge S$	Premise	(6)
$U \wedge S$	Modus ponens from (3) and (6)	(7)
S	Simplification from (7)	(8)
$P \wedge S$	Addition from (5) and (8)	(9)
$P \wedge S \to T$	*Premise*	(10)
$\therefore T$	Modus ponens from (9) and (10)	(11)
		(12)

43(15)

$\neg Q \to S$	Premise	(1)
$\neg S$	Premise	(2)
$\neg Q$	Disjunctive syllogism from (1) and (2)	(3)
$P \to Q$	Premise	(4)
$\neg P$	Modus tollens from (3) and (4)	(5)
$R \vee S$	Premise	(6)
R	Disjunctive syllogism from (2) and (6)	(7)
$\neg P \wedge R$	Addition from (5) and (7)	(8)
$\neg P \wedge R \to U$	Premise	(9)
U	Modus ponens from (8) and (9)	(10)
$\neg S \to \neg T$	Premise	(11)
$\neg T$	Modus ponens from (2) and (11)	(12)
$W \vee T$	Premise	(13)
W	Disjunctive syllogism from (12) and (13)	(14)
$\therefore U \wedge W$	Addition from from (10) and (14)	(15)

Section 2.1

11b (5) $\forall$ real numbers x such that x is positive, negative or zero.

11c (5) $\forall$ logicians x such that x is not lazy.

12b (5) $\exists$ a real number x such that x is rational.

28b (5) True

28d (5) True

28e (5) False: $x = 36$ is a counterexample.

36 (5) There is an integer n such that n^2 is even but n is not even.

Problem 3

1. (2) $\forall x,\ P(x, Kung)$
2. (2) $\forall x \exists y,\ P(x, y)$
3. (2) $\exists x \forall y,\ P(y, x)$
4. (2) $\forall x \exists y,\ \neg P(x, y)$ or $\neg \exists x \exists y,\ P(x, y)$
5. (2) $\exists y,\ \neg P(Boy, y)$
6. (2) $\exists y \forall x,\ \neg P(x, y)$
7. (2) $\exists x \forall y \forall z,\ P(x, y) \wedge (P(y, z) \rightarrow (z = x))$
8. (2) $\exists x \exists y \forall z,\ P(Noo, x) \wedge P(Noo, y) \wedge (x \neq y) \wedge (P(Noo, z) \rightarrow (z = x) \vee (z = y))$
9. (2) $\forall x,\ \neg P(x, x)$
10. (2) $\exists x \forall y,\ P(x, y) \leftrightarrow (x \neq y)$ or $\exists x \forall y \neg P(x, y) \leftrightarrow (x = y)$

Problem 4

a (2) False. $1 + 1 \neq 1 - 1$

b (2) True. $2 + 0 = 2 - 0$

c (2) False. By counterexample, $y = 1$

d (2) False. For every x, $Q(x, 2)$ is false.

e (2) True. For example $x = 1$ and $y = 1$.

f (2) True. $y = 0$

g (2) True. Again $y = 0$

h (2) False. There is no x that would make $Q(x, y)$ true.

i (2) False.