Homework 5

188 200 Discrete Mathematics and Linear Algebra

Due: — Monday : August 31, 2009: All homework will be collected at 6:00pm. Any homework submitted after that are late.

Note:

1. All homework must be submitted to the submission box.
2. Submitting homework by email is no longer allowed.

1. Problem 16 in Section 1.5 exercise. (You don't have to solve for Problem 6, just look at the solution on back of the textbook then compare it to the one obtained for Problem 16)
2. Problem 24 in Section 1.5 exercise.
3. Consider the following matrices

$$A = \begin{bmatrix} -4 & 12 \\ 1 & -3 \\ -3 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 7 & 0 \\ -4 & -6 & 5 \\ 6 & 13 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 5 & -3 & 2 \\ 0 & 4 & -9 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   For each matrix, determine if its columns form a linearly independent set. Give reasons for your answers.
4. Consider the matrices listed in problem 3, determine if the columns of each matrix span $\mathbb{R}^3$. Give reasons for your answers.
5. Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects points through the x_1 axis and then rotate them clockwise through $\pi/2$ radian.
6. Problem 9 in Section 1.8 exercise
7. Problem 22 in Section 1.8 exercise
8. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T(\overrightarrow{e_1}) = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \quad T(\overrightarrow{e_2}) = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \quad T(\overrightarrow{e_3}) = \begin{bmatrix} 0 \\ -7 \\ 5 \end{bmatrix}$$

   Determine if T is one-to-one and/or onto transformation. Justify your answers
9. Problem 10 of Section 2.2 exercise. Make sure you justify your answers.
10. Problem 12 of Section 2.3 exercise. Make sure you justify your answers.

11. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \quad B = \begin{bmatrix} a+2g & b+2h & c+3i \\ d+3g & e+3h & f+3i \\ g & h & i \end{bmatrix}, \quad C = \begin{bmatrix} g & h & i \\ 2d & 2e & 2f \\ a & b & c \end{bmatrix}$$

Suppose that $det(A) = 5$. Find $det(B)$, $det(C)$ and $det(AC)$.

12. Prove the following theorem, but do **not** use the Invertible Matrix Theorem (Theorem 8 in Section 2.3)

Theorem: If A is an invertible $n \times n$ matrix, then for each $\overrightarrow{b}$ in $\mathbb{R}^n$, the equation $A\overrightarrow{x} = \overrightarrow{b}$ has the unique solution $\overrightarrow{x} = A^{-1}\overrightarrow{b}$