Color Models

Color: human physiological and psychological responses to *light*, which is an electromagnetic wave of wavelength between 400nm to 700nm.

Pure color is produced by light with dominant frequency. See Figure 1.

Most colors are a mixture of pure colors with a spectrum spanning different frequencies. Therefore a representation of light is its spectral energy distribution.

The energy intensity of light is the area under its spectral energy distribution curve. See Figure 1.

Terminology:
Hue: dominant frequency (or pure color).
Saturation: purity of the dominant color.
Lightness: (for reflecting objects) luminance.
Brightness: (for illuminating objects) luminance.

$$luminance = energy\ intensity$$

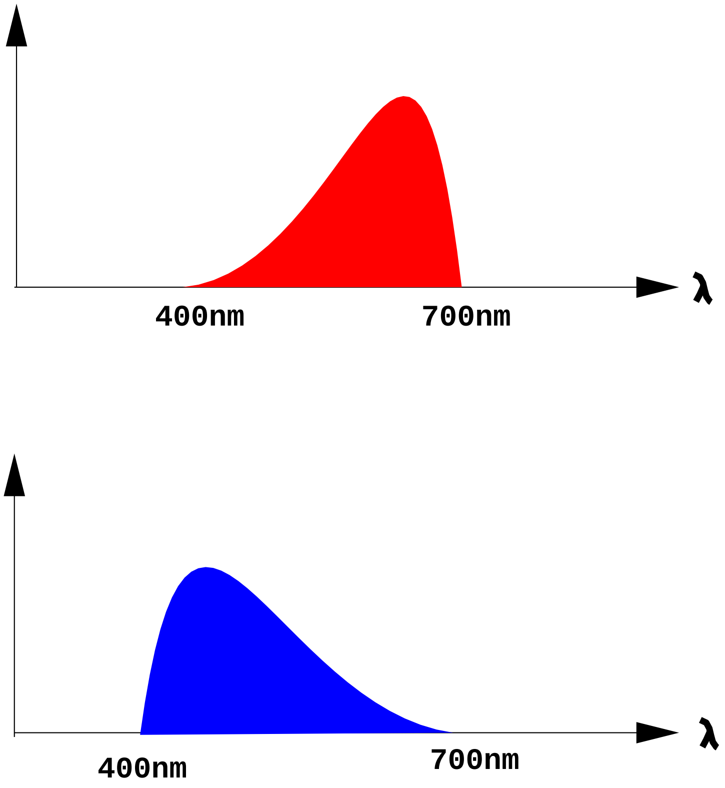


Fig. 1: Spectral Representation.

Notes:

Different spectral compositions may give rise to the same color. For example, *yellow* can be a pure color, and can also be the mixture of *green* and *red*.

It is possible to represent practical color models by a small number of parameters.

Color Models

CIE model

Three primary "colors" **X**, **Y**, and **Z** are used such that all colors in CIE space are reduced to compositions of **X**, **Y**, and **Z**.

A color C with distribution $P(\lambda)$ is represented in CIE space by

$$C = X\mathbf{X} + Y\mathbf{Y} + Z\mathbf{Z}$$

where

$$\begin{aligned} X &= k\int P(\lambda)\bar{x}(\lambda)\mathrm{d}\lambda \\ Y &= k\int P(\lambda)\bar{y}(\lambda)\mathrm{d}\lambda \\ Z &= k\int P(\lambda)\bar{z}(\lambda)\mathrm{d}\lambda \end{aligned}$$

Here, k is a constant for normalization, and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are filtering functions.

Now we get a linear color space CIE with the following property:

All visible lights are points in a cone (CIE cone) in the first octant of XYZ (CIE) space. See Figure 2.

The color (X, Y, Z) is projected through the origin into the point (x, y, z) on the $X + Y + Z = 1$ plane. See Figure 2. Points on a line passing through the origin have the same color but different brightness.

Projecting onto the X-Y plane the intersection of the CIE cone with the plane $X + Y + Z = 1$, we have the **CIE chromaticity diagram**.

Properties:

a) 100% spectrally pure colors are represented as points on the curved part of the boundary of CIE chromaticity diagram.

b) The white color (C) is near the center of the diagram. $(x = y = 1/3.)$

c) CIE chromaticity diagram does not contain the brightness component of a color.

Note that magenta is not a pure color.

The complementary color of a given color is a boundary color that can be mixed with the given color to produce white color.

Color mixing corresponds to linear interpolation in CIE space. For instance, mixing two colors with different proportions gives rise to different points on the line segment connecting the two color points in CIE diagram.

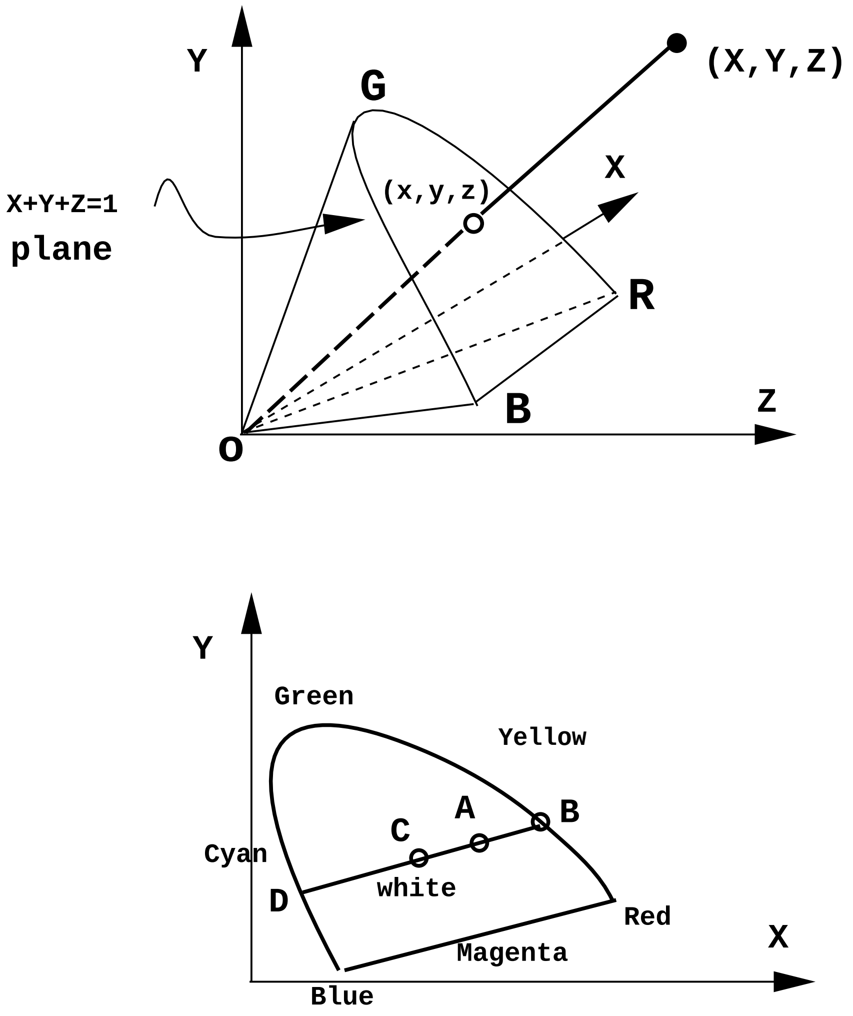
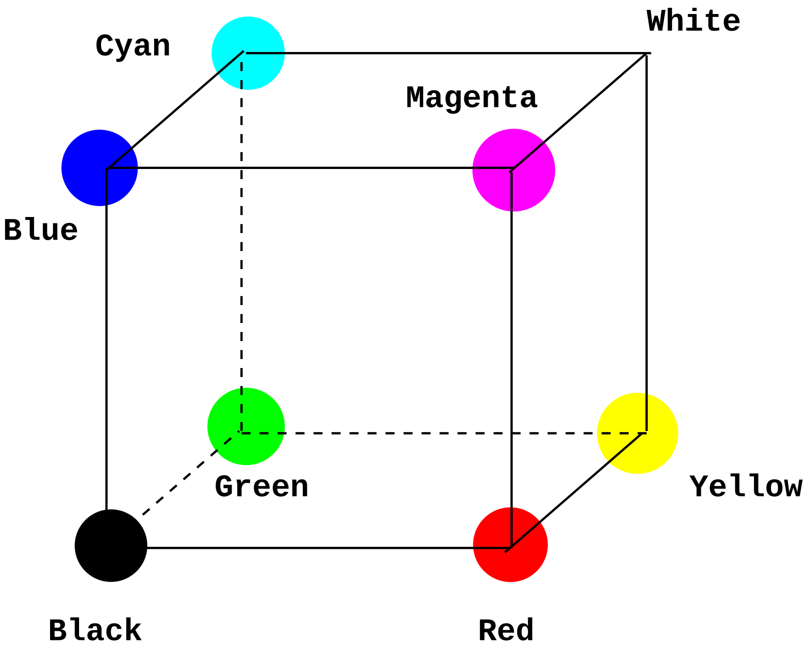


Figure 2. CIE Chromaticity Diagram.

The saturation of a color A in CIE chromaticity diagram is defined by the ratio of lengths $\overline{AC}/\overline{BC}$, where C is the white color point, B is the boundary point on the extended line CA. See Figure 2.



RGB Cube

RGB color model

The CRT of color workstation uses three primary colors, Red, Green, and Blue, to compose all displayable colors. Any color C is represented by

$$C = rR + gG + bB, \quad \text{where}(r, g, b) \in [0, 1]^3$$

See Figure 3 for an RGB cube.

Notes:

a) All displayable colors on a screen form a subset of in the 3D CIE space. This subset is a linear image of a cube.

b) This subset projects into a triangle on the 2D CIE chromaticity diagram, which is called the *realizable triangle*. Only colors in this triangle can be displayed on the screen.

The RGB model is used internally for color representation in a workstation. It is not suitable for specifying colors by users, especially when a certain color needs to be matched.

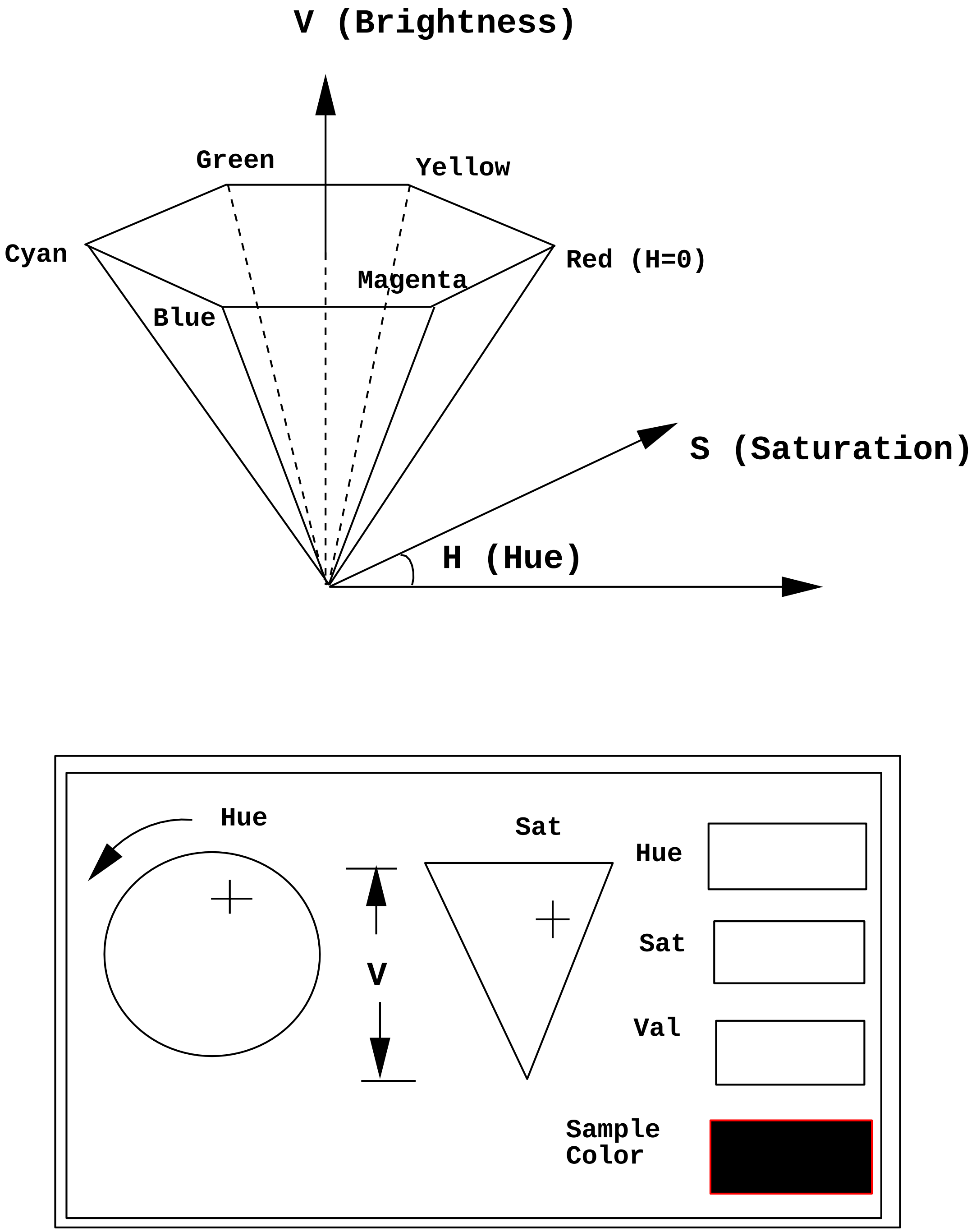


Figure 4. HSV Color Model.

HSV color model is a better color model for color specification in GUI.
(HSV = Hue, Saturation, and Value).

The HSV space is a truncated hex-cone, obtained by squeezing the top part of RGB cube along the main diagonal. See Figure 4.

a) $s = 0$ (central line) represents gray colors; in this case, $v = 1$ gives white, $v = 0$ gives black.
b) $s = 1$ and $v = 1$ represent boundary colors with full brightness.

The second diagram in Fig. 4 shows how to use HSV color model for color specification.

1) The circle is a horizontal section of HSV cone.
2) The triangle is a vertical section of HSV cone.
3) Both the circle and triangle should be properly colored to provide visual feedback.

RGB and HSV models are both for describing the color of a self-illuminating object, such as a light source or a computer screen. The composition of two colors in the RGB model amounts to adding the two lights togethers.

So RGB model is called an **additive color model**.

CMY color model for reflective surfaces.

(C = Cyan, M = Magenta, Y = Yellow.)

Consider a sheet of paper that appears yellow under white light. Because white light is a composition of lights with all frequencies of equal contributions, the paper appears yellow since all lights except for yellow light are absorbed by the surface. In other words, as

$$WhiteLight = YellowLight + BlueLight,$$

if $BlueLight$ is subtracted, then only $YellowLight$ is reflected. Therefore we have

$$Yellow = White - Blue$$

Similarly,

$$Cyan = White - Red$$

$$Magenta = White - Green$$

For example, the mixture of pigments of magenta, cyan, and yellow is black, because all wavelengths are absorbed.

CMY model is a **subtractive model**, because we may use proper amounts of cyan, magenta, and yellow pigments to control how much R, G, B components respectively remain in the light that is reflected off a surface.

Relations between CIE, RGB, HSV and CMY models

(a) RGB model and CIE model are related by a linear transformation

$$\begin{bmatrix} X \\ Y \\ B \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}.$$

(b)Between RGB and CMY, we have

$$[C\ M\ Y] = [1\ 1\ 1] - [R\ G\ B];$$

conversely,

$$[R\ G\ B] = [1\ 1\ 1] - [C\ M\ Y].$$

For instance. $(1,1,1)$ in RGB model represents white color. So the point for white color in CMY model is

$$[1\ 1\ 1] - [1\ 1\ 1] = [0\ 0\ 0],$$

(meaning not to apply any pigment to a white paper).

(c) RGB model and HSV model are not related by a linear transformation.

Questions: Which of the following statements is correct?

1. Bright colors have large saturation.
2. Increasing saturation will not affect hue.
3. The transformation from CIE space to CMY space is linear.
4. Adding red pigment to green pigment produces yellow color.